Evolutionary Approach to Portfolio Optimization

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Abstract. In this paper a portfolio optimization algorithm based on Evolution Strategies is presented. This method makes use of artificial trading experts discovered earlier by a genetic algorithm. These experts, consisting of technical analysis rules, are trained to process financial time series and to generate trading advice. Evolution Strategies lead to the optimization of portfolio structures where individual trading experts advice is integrated. This approach is tested on a sample financial time series taken from the Paris Stock Exchange. The resulting investment strategy has been compared with the Buy-and-Hold strategy and the market index. The research presented extends our previous research into stock trading.

1 Introduction

Nowadays, more and more attention is given to methods based on the principle of evolution. Evolutionary Computation has become a subject of general interest with regard to the power to solve complex optimization problems.

Evolutionary Computation has been successfully incorporated into many fields of science and technology. This paper presents an application of Evolutionary Computation within the field of financial economics to the problem of portfolio optimization, the problem being the minimization of risk attached to investments where a certain level of return is desired. Although a few analytical methods have been discovered, [18], an extension of the problem, by introducing additional risk measures, such as semivariance of return, and losing several artificial assumptions, requires a new efficient algorithm, which cannot be developed on the basis of classical methods.

There have been several attempts of using artificial intelligence methods to solve financial problems. In [3] an application of evolutionary methods to analyze the financial time series is presented. Portfolio optimization using genetic algorithms is also described in [Loraschi 1995].

The approach presented here combines the power of genetic algorithms, [5], [15], used to generate artificial trading experts. These algorithms analyze the financial time series of considered stocks against the opportunities provided by Evolution Strategies, [1], [20], which, in turn leads to the optimization of portfolio structures where individual trading experts advice is integrated.

This paper constitutes an extension of research carried out by the Artificial Intelligent Research Team of the Laboratoire des Sciences de l'Image, de l'Informatique et de la Teledetection, CNRS, ULP, Illkirch, France presented in [9].

This paper is structured in the following way: Section 2 defines the problem of portfolio optimization. Section 3 presents financial time series used in the proposed approach. In section 4, a trading process is described in detail and main assumptions and ideas are indicated. In section 5, the proposed algorithm based on Evolution Strategies is described. Section 6 presents experiments and their results. Section 7 concludes the paper and suggests some interesting extensions of the presented approach.

2 Portfolio Optimization

The main goal of investors is optimal allocation of funds among various financial assets. Searching for an optimal stock portfolio, characterized by random future returns, seems to be a difficult task and is usually formalized as a risk-minimization problem under a constraint of expected portfolio return. The portfolio risk is often measured as the variance of returns but many other risk criteria have been proposed in financial literature.

Portfolio theory may be traced back to seminal paper [14] and is presented in an elegant way in [6] or [18].

The problem considered in this paper consists of minimizing an investment risk but keeping the desired expected return. When the risk is defined as a variance of an investment, the problem can be easily solved using analytical methods. In the approach presented here, the risk is defined as a semivariance of an investment, so the standard analytical methods cannot be applied, because of the complexity of the problem.

The problem of portfolio optimization can be defined in the following way: Consider a financial market on which N risky assets are traded. Let

$$\mathbf{R}' = (R_1, R_2, \ldots, R_N)$$

be the square-integrable random vector of their returns.

Denote as $\mathbf{r} = \mathbf{E}\mathbf{R}$ the vector of expected returns and $\mathbf{V}$ the corresponding covariance matrix which is assumed positive definite.

A portfolio is a vector $\mathbf{x} \in \mathbf{R}^N$ verifying $\mathbf{x}'\mathbf{1} = 1$, where $\mathbf{1}$ is a N-component vector of ones. Hence x_i is the proportion of wealth invested in the i-th asset.

Denote as X the set of all portfolios. For each $\mathbf{x} \in X$, we define $R_x = \mathbf{x}'\mathbf{R}$ as the portfolio return and then $\mathbf{x}'\mathbf{r} = \mathbf{E}R_x$ is the portfolio expected return.

For a fixed level e of expected return, $X_e = \{\mathbf{x} \in X : \mathbf{x}'\mathbf{r} = e\}$ is the set of all portfolios leading to the desired expected return e. The optimization problem is then to find $\tilde{\mathbf{x}}$ such that:

$$\mathrm{Risk}(\tilde{\mathbf{x}}) = \min\{\mathrm{Risk}(\mathbf{x}) : \mathbf{x} \in X_e\}$$

where Risk(.) is the risk indicator (variance of returns in the Markowitz theory).

As individuals choose optimal portfolios according to their criteria, equilibrium stock prices arise on the financial market. The most celebrated equilibrium model, the so-called CAPM, was introduced by W.Sharpe, J. Lintner and J.Mossin in the mid-sixties [21], [12], [17]. The essential result is based on the perfect market assumption (no transaction costs, no restrictions on short selling and perfect divisibility of stocks) along with homogeneous expectations, meaning that investors possess the same information and interpret it in the same way.

In spite of its wide diffusion throughout the professional and academic world, the CAPM is often criticized for its artificial assumptions. Although it is an interesting theoretical model, its practical applications may often misfire.

3 Financial Time Series

In this paper, real data from the Paris Stock Exchange is considered, especially quotations of stocks belonging to its market index the CAC40. The time series contains the opening, high, low and closing prices as well as the transaction volume of each stock. Time series of the market index value (CAC40) are also available. The 40 stocks are tracked over a period of about 4 years beginning January 2, 1997.

Our approach to portfolio optimization is founded on the idea that efficient trading rules can be discovered using evolutionary methods from financial time series. These rules can form the trading experts that advise the portfolio optimization program as to when a given asset should be bought or sold. The details of the process of generating artificial trading experts is described in [8]. These experts state a part of input data for the portfolio optimization algorithm presented here.

A sample of input data containing the close prices of a few titles and the index value used in further examples presented in this paper is given in Table 2.1. The close price is used to calculate portfolio value. The return rate of the portfolio is compared with the index rate.

The real tests require a huge amount of data, thus only a brief summary of these tests will be presented in this paper.

Table 2.1. Financial Time Series - Close Prices and CAC40 Values

Date	BNP	Bouygues	France Telecom	Peugeot	Renault	TF1	CAC40
10-27-2000	94.00	60.45	125.90	207.80	53.90	63.40	6268.93
10-26-2000	94.65	59.10	119.00	211.90	51.00	67.35	6208.42
10-25-2000	96.65	58.70	123.00	214.00	50.65	67.90	6277.90
10-24-2000	96.50	58.00	121.10	209.50	50.35	69.90	6323.74
10-23-2000	94.20	54.05	113.00	208.70	48.96	66.30	6182.34
10-20-2000	93.00	55.40	107.60	204.40	49.99	65.05	6149.44
10-19-2000	93.20	54.70	103.00	203.20	48.92	65.20	6066.48
10-18-2000	92.05	51.40	98.20	201.60	48.80	63.45	5937.35
10-17-2000	92.80	51.00	102.20	201.30	49.22	63.00	6067.15
10-16-2000	91.05	47.45	102.00	205.80	49.66	61.00	6088.04
10-13-2000	91.20	46.80	101.00	208.70	49.20	57.00	6064.21
10-12-2000	89.80	47.95	98.25	211.90	49.00	57.40	5990.70
10-11-2000	89.00	47.00	96.20	207.50	49.00	59.20	5956.12
10-10-2000	95.60	51.70	103.10	211.20	51.00	60.00	6143.30
10-09-2000	96.50	50.40	108.50	216.00	51.00	55.50	6110.06
10-06-2000	100.30	51.05	115.00	214.50	51.15	60.25	6258.41
10-05-2000	99.75	54.95	120.00	214.40	51.70	63.50	6335.12
10-04-2000	99.95	55.65	120.30	211.50	52.50	63.70	6296.13
10-03-2000	99.00	57.95	125.00	211.00	51.00	64.50	6400.43
10-02-2000	102.10	56.90	123.00	210.10	50.90	62.55	6349.24
09-29-2000	99.90	57.10	121.40	201.30	48.60	65.00	6266.63

Source : Bourse-Experts Database http://www.bourse-experts.com

4 Trading Process

4.1 Artificial Trading Experts

The portfolio optimization process make use of advice provided by artificial trading experts discovered earlier by a genetic algorithm. These experts are based on technical analysis rules which assume that future trends can be identified more or less as a complicated function of past prices. Using a trading rule is a practical way of identifying trends and generating buying and selling signals.

Let S be the set of technical analysis trading rules used to take a trading decision on the market. Let M denote the cardinality of S. On the basis of past prices, each rule generates a signal: to sell, to hold or to buy. For computing simplicity these decisions will be replaced with real numbers, 0.0, 0.5 and 1.0 respectively.

In this approach, an expert $\mathbf{e} = (e_1, e_2, \ldots, e_M)$ is an M-dimensional binary vector. An i-th coordinate of the expert is equal to 1 if and only if, the expert uses the i-th rule in the decision process to generate a trading advice. Thus, there are 2^M possible experts, but only a few of them are usually efficient.

For example, $\mathbf{e} = 001101$ means that expert $\mathbf{e}$ generates advice on the basis of rules numbered 3, 4 and 6.

In order to generate expert advice, an arithmetic average $\bar{\mathbf{d}}$ of active rules decisions is calculated as follows:

$$\bar{\mathbf{d}} = \frac{\sum_{i=1}^{M} e_i \cdot d_i}{\sum_{i=1}^{M} e_i},$$

where d_i denotes the decision of the i-th rule.
Next, the obtained number $\bar{\mathbf{d}}$ is transformed to a decision, i.e. a number 0.0, 0.5 or 1.0. This can be done by means of a valuation function f and an earlier chosen threshold $s \in [0.00, 0.50]$ as follows:

$$f(\bar{\mathbf{d}}) = \begin{cases} 0.0, \text{ if } \bar{\mathbf{d}} \leq s \\ 0.5, \text{ if } s < \bar{\mathbf{d}} < 1 - s \\ 1.0, \text{ if } 1 - s \leq \bar{\mathbf{d}} \end{cases}$$

Finally, advice given by the expert is equal to $f(\bar{\mathbf{d}})$.

For example, $\mathbf{e} = 001101$, two rules lead to buy and one leads to do nothing. Hence $\bar{\mathbf{d}} = 0.8333$. The final decision is to buy the stock as long as $1-s \leq 0.8333$, where s denotes the earlier chosen threshold.

This threshold can be referred to as the risk aversion coefficient of the expert. For low levels of s the probability of doing nothing is high because the interval $[s, 1-s]$ is large. Consequently, the strategy is conservative and the expert does not transact frequently. If the opposite is true, i.e. if s is near to 0.50, almost all decisions will be to buy or to sell.

Artificial trading experts are generated daily for each stockholding in the portfolio under consideration according to the process described in [8]. These expert decisions constitute the soul of a trading process presented in next section.

4.2 Trading Process

Let $\mathbf{a}_t = (\mathbf{a}_t^{(1)}, \mathbf{a}_t^{(2)}, \ldots, \mathbf{a}_t^{(N)})$ denote the vector of expert advice for each stockholding constituting a given portfolio at the end of day t. Let $S = \{1, 2, \ldots, N\}$ and

$$S_t^{(b)} = \{i \in S : \mathbf{a}_t^{(i)} = 1.0\},$$
$$S_t^{(h)} = \{i \in S : \mathbf{a}_t^{(i)} = 0.5\},$$
$$S_t^{(s)} = \{i \in S : \mathbf{a}_t^{(i)} = 0.0\}.$$

$S_t^{(b)}$ is the set of stocks which experts advise buying, $S_t^{(h)}$ is the set of stocks which experts advise holding and $S_t^{(s)}$ is the set of stocks which experts advise selling. Superscripts $(b), (h), (s)$ are abbreviations of 'buy', 'hold' and 'sell' respectively.

At the beginning of day $t+1$, a non negative number of stocks of each stock from $S_t^{(s)}$ will be sold and a non negative number of stocks of each stock from $S_t^{(b)}$ will be bought. Let $\Delta\mathbf{x}_t = (\Delta\mathbf{x}_t^{(1)}, \Delta\mathbf{x}_t^{(2)}, \ldots, \Delta\mathbf{x}_t^{(N)})$ denote the vector made up of numbers of traded stocks.

For example, $\Delta\mathbf{x}_t = (10, 20, 0, -12)'$ means that 10 stocks of first stock are bought, 20 stocks of second stock are bought and 12 stocks of fourth stock are sold.

The following constraints should of course be fulfilled:

$$\begin{aligned} \Delta\mathbf{x}_t^{(i)} &> 0, \qquad \text{for } i \in S_t^{(b)}, \\ \Delta\mathbf{x}_t^{(i)} &= 0, \qquad \text{for } i \in S_t^{(h)}, \\ \Delta\mathbf{x}_t^{(i)} &< 0, \qquad \text{for } i \in S_t^{(s)}. \end{aligned}$$

Moreover, a budget constraint, as presented below, should be fulfilled.

$$\sum_{i \in S_t^{(b)}} (1+c) \cdot \mathbf{p}_{t+1}^{(i)} \cdot \Delta\mathbf{x}_t^{(i)} \approx \sum_{i \in S_t^{(s)}} (1-c) \cdot \mathbf{p}_{t+1}^{(i)} \cdot \Delta\mathbf{x}_t^{(i)},$$

where $\mathbf{p}_t = (\mathbf{p}_t^{(1)}, \mathbf{p}_t^{(2)}, \ldots, \mathbf{p}_t^{(N)})$ denotes the vector of opening prices at day t and c stands for a transaction cost.
This condition comes from the idea of self financing, which is discussed in the next section.

The process begins with a portfolio $\mathbf{x}_0$ at time t_0. Let $X^{(1)}$ be a space consisting of all portfolios which can be obtained from $\mathbf{x}_0$ at time t_1 according to the process presented above. The purpose is to find a portfolio $\mathbf{x}_1 \in X^{(1)}$ minimizing the risk factor (i.e. semivariance) within the space $X^{(1)}$. By repeating this process, a sequence of trading decisions, which constitutes an investor strategy, can be obtained.

4.3 Idea of Self Financing

One of the main assumptions in this approach is the idea of self financing. All funds are invested at the beginning of the trading process and, while the process is running, funds can neither be added nor withdrawn. However, a small amount of money can be used to fulfill the equality as defined in the previous section.

This amount of money is needed, because of the fact that trading decisions are taken at a time, when the stock price is unknown. All calculations are made after the market is closed. This means that the decisions made can be realized the next trading day when the market is open. At the time these decisions are made, the next day's opening prices are unknown and they are approximated by the previous day's closing prices.

The important question is what to do in the case where $S_t^{(b)} = \emptyset$ or $S_t^{(s)} = \emptyset$. If $S_t^{(b)} = \emptyset$, nothing is done, because the obtained funds cannot be invested elsewhere. Similarly, if $S_t^{(s)} = \emptyset$, there will be no trading, because no funds are obtained.

A special risk-free asset can be introduced which allows funds obtained in selling operations to be stored, and making it possible to realize buying operations where there has been luck in selling transactions. In order to avoid a situation where all the funds of the risk-free asset are invested on the first possible date, a threshold is defined, which limits the percentage of funds available for investing on one day.

5 Evolution Strategy

The process described in the previous section is optimized using an evolutionary algorithm based on Evolution Strategies, described in detail in [19] and [20]. In this section, the modifications introduced to the standard algorithm are presented.

In this approach, a portfolio is encoded as a real valued vector of dimension N, where N denotes the number of stocks included in the portfolio.

To evaluate the portfolios generated during the process of evolution, various objective functions can be used. In the prototype designed here, several functions, based on expected return and risk factors, are implemented. The available objective functions are the following:

$$F_1(\mathbf{x}) = \frac{1}{1 + \varepsilon_1 \cdot \mathrm{SVar}(R_x)},$$

$$F_2(\mathbf{x}) = \frac{1}{1 + \varepsilon_1 \cdot \mathrm{SVar}(R_x) + \varepsilon_2 \cdot |\beta_x - \beta_{x_0}|},$$

$$F_3(\mathbf{x}) = \frac{1}{1 + \varepsilon_1 \cdot \mathrm{Cov}(R_x, R_i) + \varepsilon_2 \cdot |\beta_x - \beta_{x_0}|},$$

$$F_4(\mathbf{x}) = \frac{1}{1 + \varepsilon_1 \cdot \mathrm{SVar}(R_x) + \varepsilon_2 \cdot \mathrm{Cov}(R_x, R_i) + \varepsilon_3 \cdot |\beta_x - \beta_{x_0}|},$$

where $\mathbf{x}_0$ denotes the initial portfolio, given by the user, R_i stands for the market return and β_x, β_{x_0} stand for the β coefficient of the considered portfolio $\mathbf{x}$ and the initial portfolio $\mathbf{x}_0$ respectively.
Factors $\varepsilon_1, \varepsilon_2$ and ε_3 are used to tune the algorithm and to adjust the importance of each component of the objective function. The objective functions refer to some heuristics using parameters such as the β coefficient. By introducing the difference between the β_x of the generated portfolio and the β_{x_0} of the portfolio of reference, we penalize the portfolio having β_x far away from β_{x_0} of the reference. Nevertheless, the performance of a solution is defined in terms of expected return and risk of the portfolio on a test period as mentioned in previous sections.

Several methods of generating an initial population are used. The simplest method is random generating with uniform probability. It consists of μ-times random choosing of an individual from the search space. The probability of choosing an individual is the same for every individual in the search space.

The second method uses an initial portfolio given by a user as an algorithm parameter. An initial population is chosen from the neighborhood of the given portfolio. This is done by generating a population of random modifications of the initial solution.

Every individual in the initial population has to meet the financial constraints. Thus, after random generation, every individual undergoes a validation process. If it is not accepted, it is replaced with another random-generated individual. In this way, the initial population is perfected, which means that it satisfies all the desired conditions.

In the algorithm, common evolution operators such as reproduction and replacement are used.

In the process of reproduction, a population of size μ generates λ descendants. Each descendant is created from ρ ancestors. Reproduction consists of three parts: parent selection, recombination and mutation, repeated λ times.

Parent selection consists of choosing ρ parents from a population of size μ. There are several commonly used methods of parent selection. The simplest method is random choosing with uniform probability. One of the most popular methods is random choosing using the "roulette wheel", which means that the probability of choosing an individual is proportional to its value of the objective function.

Recombination consists of generating one descendant from ρ parents chosen earlier. The recombination operators described in the previous section, such as no recombination, global intermediary recombination, local intermediary recombination, and uniform crossover, are incorporated into the system.

The approach uses a self-adaptive mutation which is presented in [2] and [20]. The parameters of the mutation are encoded in an individual together with a representation of the portfolio.

Each generated descendant has to undergo a process of verification in order to satisfy several constraints. An individual is accepted if the portfolio that it represents can be obtained in accordance with the trading process from the initial portfolio. Otherwise, the individual is rejected, and the process of reproduction is repeated. As a result of this verification, offspring are obtained according to the trading process and the idea of self-financing is fulfilled.

In the replacement process, a new population of size μ is chosen from an old population of size μ and its λ descendants.

The simplest method of replacement is deterministic selection. According to this method, from $(\mu + \lambda)$ individuals, i.e. from the union of an old population and its offspring, μ best survivals are chosen. But every individual can survive no more than κ generations in history.

Apart from deterministic selection, the tournament selection can be used. To start with, τ individuals are randomly chosen from the union of an old population and its offspring. From these τ individuals, the best one is chosen for the new population. By repeating this process μ times a new population is obtained.

Termination criteria include several conditions. The first condition is defined by the acceptable level of valuation function value. The second is based on the homogeneity of population, defined as a minimal difference between the best and the worst portfolio. The third condition is defined as a maximal number of generations. The algorithm stops when one of them is satisfied.

6 Experiments

All experiments have been carried out using the *EPO* (Evolutionary Portfolio Optimizer) system described in details in [13]. The artificial trading experts required have been generated by *ACT* system presented in [8].

Two general types of tests have been carried out. The first one refers to a portfolio made up of 10 stocks randomly chosen among the stocks of the CAC40 index. The purpose of the test was to evaluate the algorithm efficiency for medium portfolios. The second type of tests refers to a portfolio consisting of all 40 stocks of the CAC40. The purpose of this was to compare the performance of each computed portfolio with that of the market portfolio approximated by the index return.

Each test was repeated several times during different time periods to avoid bias. In addition, various initial portfolios were used.

By selecting an initial portfolio and carrying out evaluations over the test period for each day of the test period, the optimal portfolio was discovered. The portfolio calculated for the next day would have been the optimal one, according to the constraints defined by expert decisions and the principle of self-financing. Table 5.1. presents an example of such investment strategy. Moreover, according to the heuristics, the β coefficient of these portfolios will be relatively stable as compared to its initial value. In addition, the performance of the result was evaluated on the basis of expected return and risk, the latter being defined as the semivariance of the portfolio return. Finally, the profit obtained in the suggested trading process had a significant impact.

Table 5.1. Simulation

Date	BNP	Bouygues	France Telecom	Peugeot	Renault	TF1
09-29-2000	100	100	100	100	100	100
10-02-2000	182	80	73	88	174	19
10-03-2000	178	53	43	74	277	67
10-04-2000	148	66	7	91	395	28
10-05-2000	229	113	31	82	110	63
10-06-2000	224	45	67	91	94	47
10-09-2000	236	136	37	84	93	20
10-10-2000	178	65	70	56	323	17
10-11-2000	69	79	34	115	337	22
10-12-2000	215	147	18	72	185	46
10-13-2000	185	28	65	66	301	35

Source: Results obtained using EPO system.

Initial portfolios as well as test periods were randomly chosen. To begin with, 10 stocks were selected from these 40 making up the CAC40. Next, an initial date (i.e. the start of the test period) was set. After that, an initial portfolio was randomly generated. To check the efficiency of the approach more closely, the experiment was repeated several time. Each time a different initial population was chosen. Moreover, the whole experiment was repeated for a different initial date.

In all experiments, the length of a test period was equal to 20 or 60 days. It is worth noticing that the length of a test period had no influence on the algorithm quality because the computation was carried out separately for each day. There was no difference whether a period of 60 days was considered at once or three separate periods of 20 days.

In order to assess the results, the final profit was compared with the profit achieved by the Buy&Hold (B&H) strategy, which consists of keeping the initial portfolio unchanged during the whole test period. In most cases, the suggested strategy outperforms the simple B&H strategy. Although tightly linked to the test period and current trends of the market, repeating these experiments several times on different test periods confirmed the quality and the efficiency of the proposed approach. The summary of obtained results is presented in the table 5.2.

Table 5.2. Summary of Results

Stocks	Length of test period	Number of tests	Number of results outperforming B&H	Number of results outperforming index
10	20 days	10	9	2
10	20 days	10	10	1
10	20 days	10	6	10
10	20 days	10	7	10
10	60 days	10	8	0
10	60 days	10	10	10
40	20 days	10	7	1
40	60 days	10	4	0

Source: Results obtained using EPO system.

It is worth noting that the quality of the obtained results depends on the quality of the artificial trading experts generated earlier (see the detailed study of expert performance in [8]). Moreover, the current market situation is also important because when the prices of a large number of stocks are increasing, the results obtained are usually satisfactory, but they do not outperform the Buy&Hold strategy. In the inverse case, when prices of a large number of stocks are decreasing, the obtained profit will not be very high, but it is usually higher than the profit of the Buy&Hold strategy. In order to appreciate the true quality and effectiveness of performance more experiments on various financial markets would be needed, although first test results are very promising.

7 Conclusions and Perspectives

This thesis presents the evolutionary approach to the problem of portfolio optimization. The goals and constraints of the problem have been presented and an algorithm based on Evolution Strategies has been proposed. This approach rejects some artificial assumptions used in theoretical models (such as perfect divisibility of stocks); it introduces transaction costs and alternative risk measures such as the semivariance. The approach has been evaluated and validated using real data from the Paris Stock Exchange.

In order to evaluate this approach, the resulting investment strategy has been compared with the Buy-and-Hold strategy. To reduce the time period bias on performance, several time series were selected. The results demonstrate that the evolutionary approach is capable of investing more efficiently than the simple Buy&Hold strategy.

The algorithm presented can be further improved by modifying evolutionary operators, especially recombination. The fitness function study can also increase the efficiency of the method. Moreover, the application of special random number generators seems to be reasonable. Additional effort should be spent on methods of portfolio validation in order to eliminate unacceptable solutions at the moment of its creation.

The evolutionary approach in stock trading is still in an experimental phase. Further research is needed, not only to build a solid theoretical foundation in knowledge discovery applied to financial time series, but also to implement an efficient validation model for real data. The approach presented seems to constitute a practical alternative to classical theoretical models.

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