Zastosowania rozwiązań informatycznych w bankowości
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PORTFOLIO OPTIMIZATION USING DIFFERENTIAL EVOLUTION

1. Introduction

In the field of financial economics, portfolio choice theory can be traced back to the seminal work of Markowitz [14]. In this framework, agents are supposed to minimize the variance of the return of their portfolio, given a desired expected return. They then solve an optimization problem in the mean-variance space. The set of optimal portfolios when the expected return varies is called the efficient frontier. When introducing a risk-free asset and some equilibrium considerations, one gets the so-called Capital Asset Pricing Model (referred to as CAPM in the following) developed by Sharpe [19], Lintner [10] and Mossin [15]. This model gives the equilibrium relationship between the expected return of a security with respect to the expected return of the market portfolio. It also says that every agent possesses the market portfolio independently of his attitude toward risk. Risk aversion enters the story only to determine which proportion of wealth has to be invested in this optimal portfolio. This approach has been questioned in several points, especially concerning the question of the mean-variance efficiency of the benchmark portfolio used in empirical studies, for example a stock index, equally or value weighted. Roll [18] is a seminal reference on this subject but many authors have elaborated contributions in this vein. In fact, the efficient frontier depends on the probability distribution of returns; consequently, when there are information assymetries, the efficient frontier may vary from one agent to another (see for example Dybvig-Ross [4; 5]).

In order to evaluate whether a managed portfolio generates superior returns, it is fundamental to adjust the portfolio return to its risk. Many adjustments procedures were developed in the sixties, the most celebrated being Sharpe's ratio [20], Jensen's alpha and Treynor's ratio. The first one measures risk by the total variance of returns while the others are based on the CAPM, taking the portfolio as the risk measure to be the corresponding β coefficient.

Evaluating the ability of a portfolio manager to generate superior returns is a difficult task because the composition of the portfolio and its evolution through time are not always known; this is the case in most studies related to mutual fund performance (see [8] for a survey). However, when testing a new method of portfolio management, as in the present paper, the successive portfolios are known and the criterion of the positive weighting measure can be used [7].

In this paper a new approach to dynamic portfolio management is proposed, integrating the classical approach of using variance of returns and β coefficient in the objective function, genetic algorithms and differential evolution.

Since K. Price attempted to solve the Chebyshev polynomial fitting problem using the evolutionary approach [16], differential evolution has been applied in many domains, particularly in applications where the space of solutions is non linear and non differentiable. Briefly speaking, differential evolution can be considered as a population based, stochastic function optimizer. The idea of optimization is founded on a strategy that generates variations of the parameters guided by the objective function. Vector variations are obtained through the application of genetic operators to the members of successive populations. Resulting from the process of evolution, the best vector represents the final solution. In this paper, this new approach applied to portfolio optimization is presented and illustrated in the framework of portfolio management with stock market time series. This research is strongly related to our previous paper on stock timing using genetic algorithms [11].

2. A quick overview of portfolio theory

In this section, the essential results of the classical portfolio theory are recalled; well-known proofs which may be found in standard textbooks like Huang-Litzenberger [9] or Roger [17] are not provided.

2.1. The mean-variance efficient frontier

Consider a financial market with N risky assets and denote as $\tilde{r}' = (\tilde{r}^1, \ldots, \tilde{r}^N)$ the vector of returns[1]. $\mathbf{V}$ is the covariance matrix of returns and $\bar{\mathbf{r}} = E[\tilde{r}]$ denotes

[1] As usual, a prime stands for transposition.

the corresponding vector of expected returns. A portfolio is a vector $x'=(x^1,...,x^N)$ such that $\Sigma x^k = 1$, x^k being the proportion of wealth invested in the k-th asset. Denoting by $R_x = \Sigma x^k \widetilde{r}^k$ the portfolio return, and fixing a level e for the desired expected return, the optimal choice is the solution of the following problem:

$$\min_y 1/2y'\mathbf{V}y$$

under the constraints

$$y'\bar{\mathbf{r}} = e$$

$$y'\mathbf{1} = 1 \tag{1}$$

where **1** is a vector the components of which being equal to 1. The first constraint defines the expected return of the portfolio as desired by the investor while the second is the so-called portfolio constraint meaning simply that no more than 100% of wealth is invested. However, no positivity constraint is imposed on the y^k meaning that short-selling is allowed.

The optimal solution is given by:

$$x^* = \frac{1}{D}\left[(eC - A)\mathbf{V}^{-1}\bar{\mathbf{r}} + (B - eA)\mathbf{V}^{-1}\mathbf{1}\right]$$

with the following notations:

$$A = \bar{\mathbf{r}}'\mathbf{V}^{-1}\mathbf{1}$$

$$B = \bar{\mathbf{r}}'\mathbf{V}^{-1}\widetilde{\mathbf{r}}$$

$$C = \mathbf{1}'\mathbf{V}^{-1}\mathbf{1}$$

$$D = BC - A^2$$

When $e = E(\widetilde{R}_{x^*})$ varies, we obtain the efficient frontier which relates the optimal expected returns to corresponding variances. The equation of the efficient frontier is given by:

$$C\sigma^2_{x^*} - \frac{C^2}{D}(E[\widetilde{R}_{x^*}] - \frac{A}{C})^2 = 1$$

where $\sigma^2_{x^*}$ is the variance of returns of the portfolio x^*. Usually, efficient portfolios are represented in a two-dimensional space with the standard deviation σ_{x^*} on the horizontal axis and the expected return $E(\widetilde{R}_{x^*})$ on the vertical one. We obtain a hyperbola (see Fig. 1) and only the upper-half can be optimal for the investors, while the lower half contains dominated portfolios (lower expected returns and greater variances).

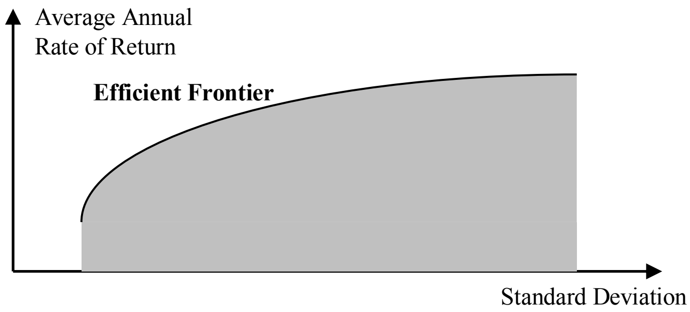

Fig. 1. Efficient frontier in the σ-E space

2.2. The Capital Asset Pricing Model

When a risk-free asset is introduced, for example the asset numbered 0, it can be shown that investors share their wealth between a portfolio of risky assets and the risk-free security. Moreover, at equilibrium, they choose the same portfolio of risky assets, the so-called market portfolio.

The optimal risky portfolio for all agents is given by:

$$x^m = \frac{\mathbf{V}^{-1}(\bar{\mathbf{r}} - r^0\mathbf{1})}{\mathbf{1}'\mathbf{V}^{-1}(\bar{\mathbf{r}} - r^0\mathbf{1})}.$$

The attitude of agents toward risk only enters the story to determine which proportion is invested in the risk-free asset as opposed to the risky portfolio. Figure 2 illustrates this result. The line joining x and the risk-free rate (denoted as $r_\$$ on the figure) is the set of optimal portfolios available to the investors. In fact every other line joining $r_\$$ and a portfolio inside the hyperbola is dominated by a portfolio of the tangent line. Consequently, this tangent portfolio is chosen by all the investors as their portfolio of risky assets.

Moreover, the following relationship is obtained:

$$E(\tilde{r}^k) - r^0 = \beta_k\left(E\left(R^m\right) - r^0\right) \tag{2}$$

where R^m denotes the return of the market portfolio, $E(\tilde{r}^k)$ is the expected return on asset k and $\beta_k = \dfrac{\operatorname{cov}(\tilde{r}^k, R^m)}{V(R^m)}$.

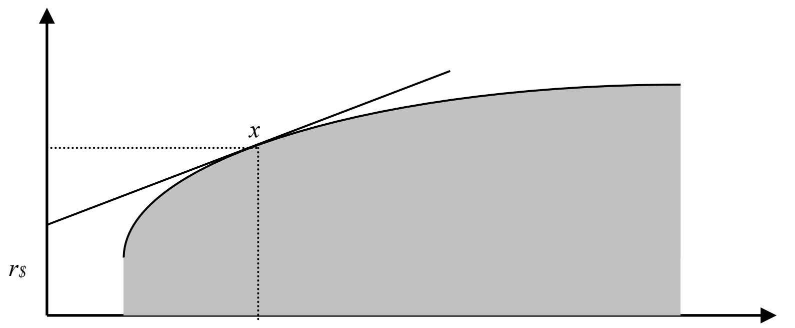

Fig. 2. The efficient frontier with a risk-free asset

The preceding results show that, in theory the portfolio choice problem can easily be solved and it possesses an analytical solution. This, however, relies on restrictive assumptions regarding either the preferences of agents or the distributional properties of returns. Moreover, expected returns and the covariance matrix would have to be identical among the population of agents. This is surely not the case in practice because different portfolio managers have different expectations. Finally, the market portfolio is not observable and a proxy is used in practical studies. All these considerations lead to the proposal of an alternative way to deal with the choice of an optimal portfolio. Before presenting the differential evolution approach, some insights into the benchmark problem are given below, i.e. the problem of selecting a proxy for the market portfolio.

2.3. The benchmark problem

The market portfolio is not known because it theoretically contains all financial assets, stocks, bonds, real estate, etc.. In empirical studies, a widely used proxy is a stock index like the S&P500 index for the US market or the CAC40 index for the French market. Roll [18], in a celebrated paper, argued that choosing such an index is not satisfactory because there is no reason for the index to lie on the mean-variance efficient frontier. Moreover, when two agents have different information about financial securities, they build different efficient frontiers because their parameters ($\mathbf{V}$ and $\bar{\mathbf{r}}$) are not the same.

Last but not least, the composition of a managed portfolio evolves through time due to successive adjustments while the classical theory is essentially a static approach. Consequently, testing the performance of a portfolio strategy is not an easy task. In the following, we use the so called positive period weighting measure to test if our genetic approach exhibits a specific ability to choose among stocks. This criterion, due to Grinblatt-Titman [7], is based on the following principle. Consider a time-series of daily portfolio returns $R_t,\ t=1,...,T$ and the corresponding time-series of returns for the benchmark portfolio $R_t^B,\ t=1,...,T$. If the portfolio manager has superior ability in forecasting the returns, there exists a vector $\gamma \in \Re^T_+$ of weights summing to 1 such that:

$$\sum \gamma_t R_t > 0$$
$$\sum \gamma_t R_t^B = 0$$

The intuition behind this criterion is the following: assume that the weights are the marginal utilities of an investor who holds the benchmark portfolio. When $\sum \gamma_t R_t > 0$, this quantity may be interpreted as the improvement of the expected utility of the agent when he adds the managed portfolio to its initial one.

3. Dynamic strategies

In the real world, portfolios are dynamically managed. Our adjustment process for each period of time, i.e., on a daily basis, will be now described. The notations used are the same as in the preceding section. However, more detail is required because the management of a portfolio involves buying quantities of assets at specified prices; it is not sufficient to work with investment proportions only. Moreover, when assets are traded, transaction costs are incurred which penalize the final return. It is therefore necessary to carefully describe the trading process to be able to evaluate the performance of the portfolio or, more precisely, the sequence of portfolios.

For the sake of simplicity, the risk-free asset will be numbered 0 and the risky assets from 1 to *N*. For each security, we build a genetic expert which is able to give three types of advice for each security: buy, do nothing or sell. The set of advice given by the *N* experts at the end of day *t* is represented by a vector $a_t = \left(a_t^1, ..., a_t^N\right)$ where:

$$a_t^k = \begin{cases} 1 \text{ for a purchase,} \\ 0.5 \text{ for doing nothing,} \\ 0 \text{ for a sale.} \end{cases}$$

The advice given is the result of a genetic procedure described in our previous work [11]. No decision is taken concerning the risk-free asset because, in our approach, its role is only to fulfil the budget constraint (see section 4).

Let $p_t' = \left(p_t^1, p_t^2, ..., p_t^N\right)$ denote the vector of opening prices at date *t,* and $\pi_t' = \left(\pi_t^1, \pi_t^2, ..., \pi_t^N\right)$ denote the vector of corresponding closing prices. The distinction between the two is important in our framework because we assume that each a_t is defined by using the information contained in past and present prices, up to the closing price of date *t.* In reality, trades, determined by the a_t, are executed at opening prices of the following day, that is p_{t+1}. This rule is considered to take into account real market conditions because it is then implicitly assumed that closing and opening prices for two consecutive days are different, which is actually the case on the French market.

We denote the portfolio possessed during day $t-1$ by $\theta_t \in \Re^{N+1}$ (asset numbered 0 is the risk-free asset); it allows trading to begin with an initial endowment θ_0 before the first market session at date 0. It is worth noticing that the θ are quantities, not proportions.

While we want to compare performances of different strategies, it is easier to begin with the same portfolio θ_0 for all strategies (the benchmark). The first portfolio adjustment resulting from transactions on the financial market is then θ_1. $\theta_t \in \Re^{N+1}$ and θ_t^0 gives the number of units of a risk-free asset. The price of the risk-free asset is fixed to one; as we consider a short horizon (one or two weeks), we neglect the discounting process and assume that the price of the risk-free asset is constant. In fact, when the cash level is positive, it could be invested at a positive risk-free rate but within the very short term which we are dealing with, the wealth difference would be negligible.

4. The trading process

In order to compare different strategies, it is reasonable to normalize the trading processes in such a way that the strategies are self-financed. This is possible by assuming that prices are known at the time of trading although this is not the case in real markets, even if conditions of trade execution are specified. In this paper, the genetic experts decide to sell or buy securities after the closing of the market and execute trades at the opening on the next day. As a consequence, $\pi_t' \neq p_{t+1}$ it may then happen that portfolio adjustments are only approximately self-financed. This is the essential reason that explains the use of a risk-free asset. In other words, as in the CAPM philosophy, the agent is allowed to choose the proportion to invest in the risk-free asset, depending on risk aversion, and only the composition of the optimal risky portfolio is considered. The transaction costs are supposed to be proportional (with a rate denoted as c), constant for all assets. In numerical experiments, we consider c = 0.2%, which seems reasonable for liquid stocks trading.

4.1. The budget constraint

In a first approach, a benchmark is defined against which the genetic experts will „fight"; in other words the performance of the dynamic portfolio is compared with that of the buy-and-hold strategy. As the latter does not necessitate adjustments between 0 and T, it would not be necessary to invest in the risk-free asset. Comparing performances on a fair basis requires that the two initial portfolios are defined in the same way. It is then assumed that the initial wealth is equally invested in the N risky assets for the two portfolios. At the terminal date T, the performance of the buy and hold strategy is defined as:

$$r^{bh} = ln\left(\frac{V_t^{bh}}{V_0^{bh}}\right)$$

where V_t^{bh} is the value of the buy and hold strategy at time t. This then gives:

$$V_0^{bh} = (1-c)\sum \theta_1^i p_1^i, \qquad i = 0, 1, 2, \ldots, N$$

$$V_T^{bh} = \sum \theta_0^i \pi_T^i, \qquad i = 0, 1, 2, \ldots, N$$

with θ_1^i as the number of units of asset i bought at date 0. While the number of stocks must be an integer, it may happen that θ_1^0 is not equal to zero even if the aim is to find only the optimal risky portfolio. The second relationship means that no transaction costs are borne at the end of the period. This is not a restrictive assumption as long as it is applied to the two portfolios.

The initial value of the dynamic (genetic) strategy, denoted as V_0^g is also equal to V_0^{bh}; in the following periods, the variation of the value process is defined as:

$$V_{t+1}(\theta) - V_t(\theta) = \sum_{i=0}^{N} \theta_{t+1}^i p_{t+1}^i - \sum_{i=0}^{N} \theta_t^i p_t^i$$

with the constraints:

$$\sum_{i=0}^{N} (\theta_{t+1}^i - \theta_t^i) p_t^i (1+c) \mathbf{1}_{\{\theta_{t+1}^i - \theta_t^i > 0\}} = -\sum_{i=0}^{N} (\theta_{t+1}^i - \theta_t^i) p_t^i (1-c) \mathbf{1}_{\{\theta_{t+1}^i - \theta_t^i < 0\}} \qquad (3)$$

The l.h.s of equation (3) values the cash-outflow generated by buying at date t while the r.h.s is the corresponding cash-inflow linked to the selling of assets.

4.2. The portfolio adjustment process

Let $S = \{1; 2; \ldots; N\}$; we define the subsets S_t^b, S_t^n, S_t^s in the following way:

$$S_t^b = \{j \in S \mid a_t^j = 1\}$$
$$S_t^n = \{j \in S \mid a_t^j = 0.5\}$$
$$S_t^s = \{j \in S \mid a_t^j = 0\}$$

where superscripts b, n, s mean „buying", „doing nothing" and „selling", respectively.

For the strategy to be self-financed, it is necessary that the following equality approximately holds:

$$\sum_{i \in S_t^b} p_t^i (1+c) \Delta\theta_{t+1}^i \approx \sum_{i \in S_t^s} p_t^i (1-c) \Delta\theta_{t+1}^i \quad (4)$$

where $\Delta\theta_{t+1}^i = \theta_{t+1}^i - \theta_t^i$. It is also obvious that $\Delta\theta_{t+1}^i \geq 0$ if $t \in S_t^b$. This first reallocation rule means that a simple transfer is realized between stocks which are to be sold and stocks which are to be bought ; it means that no change occurs for the portfolio of securities belonging to S_t^n. Equation (4) is a rewriting of equation (3) but it has been put in an approximate equality; in fact, quantities to be bought or sold are decided at the market close of day $t-1$ but trades are executed at the next day opening and prices have changed in the meantime. The difference between the two members of equation (4) determines the new amount of cash.

Table 1 gives an example of such an adjustment. The value of the portfolio before adjustment is equal to:

$$V_{t-1}^g = \sum_{i=0}^{N} \theta_{t-1}^i \pi_{t-1}^i = 26100 .$$

The amount of sales is equal to 14 305 × 0.998 = 14 276; 71 units of the first asset and 59 units of the second one are bought, leading to a cash-outflow (valued at opening prices) of (7171 + 6962) × 1.002 = 14161. The difference, equal to 116, is invested in the risk-free asset. It must be mentioned that the gap between the investments in the two first assets comes from the change in prices between *t-1* and *t*. In fact we have *Ent*(14305/(2 × 100 × 1.002))=71 and *Ent*(14 305/(2 × 120 × × 1.002)) = 59. The coefficient 0.5 comes from the limitation of variations in holdings, fixed to 50%.

Table 1. The adjustment process: an example

a_{t-1}^{i}	π_{t-1}	θ_{t-1}^{i}	p_{t}^{i}	θ_{t-1}^{i}
Buy	100	20	101	91
Buy	120	40	118	99
Do nothing	50	100	50.5	100
Sell	200	50	199	0
Sell	30	10	30.5	0
Sell	40	100	40.5	0

In other words, a relatively prudent strategy in the portfolio's rebalancement process has been considered. More aggressive behavior, however, may be adopted by following experts' advice entirely. This would lead to increased transaction costs and decreased diversification.

Some special situations may arise in the adjustment process; for example, it may be the case that S_t^b or/and S_t^s are empty. In this case, a special decision has to be taken in the following way:

$S_t^b = \varnothing$ and $S_t^s \neq \varnothing$ $\Rightarrow$ sell S_t^s and invest in the risk-free asset,

$S_t^s = \varnothing$ $\Rightarrow$ do nothing or invest available cash in S_t^b stocks.

Other rules could, however, be implemented. If is accepted that the experts' advice is reliable, a more aggressive strategy might be to sell the stocks in set S_t^n to invest in S_t^b. This, however, would increase the risk of the portfolio by making it less diversified.

5. The differential evolution approach

Even if one can prove through numerous tests that a „genetic" portfolio outperforms a buy-and-hold strategy, the problem of risk remains still unsolved. In fact, as mentioned above, performance analysis must take into account the risk-component of the strategy. It means that two strategies, to be comparable, must bear approximately the same risk.

As a first step, a test of the genetic approach can easily be implemented by building sequences of portfolios with an approximately constant β, equal to the β of the initial benchmark portfolio. For example, as a starting point, the market portfolio or an index which is used as a proxy can be taken and the weights of the selected assets could be constrained to maintain the β near 1. With the passage of time, the initial portfolio would be modified by the genetic experts and sub-optimal diversification would appear, meaning that not only systematic risk but also specific risk would be borne by the investor. To take into account these two types of risk, an objective function depending on both the β coefficient and the variance of portfolio return will be defined.

5.1. The basics of differential evolution

The basic ideas of differential evolution can be found in [16; 21]. In this paper, the specification of differential evolution is adapted to the problem of portfolio optimization. The general schema of differential evolution describes the following algorithm:

$t = 0; n = \mu$ /* t population number, n number of portfolios in the population */
Initialization x_t /* initial population of portfolios $x_{ti} = (x_{t_1}, x_{t_2}, ..., x_{t_k})$ */
/* where each portfolio is encoded as a chromosome $(v_{i,1}, v_{i,2}, ..., v_{i,m})$ */
Evaluation $P(t)$ /* computing of fitness function $\Phi(x_j(t))$ */
While stopping criteria are not reached **do**
$t = t + 1$
x_{t-1} = ParentSelection (x_{t-1})
$x_t^{'}$ = RecombinationByMutation (x_{t-1})
$x_t^{'}$ = ReplacementStrategy (x_t)
/* Strategy $(\mu + \lambda) \rightarrow x_t = x_{t-1} \cup x_t^{'}$ */
/* Strategy $(\mu, \lambda) \rightarrow x_t = x_t^{'}$ */
End While
Return the best portfolio

As we know, each portfolio is composed of m securities $v_{i,1}, v_{i,2}, ..., v_{i,m}$. The initial partition can be randomly generated or given by the user. In the algorithm, the portfolio composition is usually known in advance. This means that a user already disposes a certain number of securities or refer to the reference portfolio (e.g. CAC40). In this case, an initial population is generated as a set of portfolios around the initial portfolio, usually via a gaussian perturbation. The initial population of portfolios is generated by random modification of the values of the portfolio of reference. In fact, each portfolio should have β close to the β_* of the portfolio of reference. In our system, the acceptable values for β are given by the user.

The number of various compositions of portfolio is enormous. In order to find the optimal solution, the evolution process is guided by a fitness function which makes possible to evaluate each portfolio in the population. In general, finding a pertinent evaluation function is crucial to the optimization process. This task is particularly difficult in the domain of portfolio optimization because of multiple conflicting criteria among which might be various user preferences and multiple non-linear constraints.

In this paper the portfolio evaluation function is based on the portfolio coefficient β computed from the time series of securities, and on the variance of return:

$$F(x_i) = \frac{1}{1+\alpha\times\sigma_{xi}^2+\gamma\times\left|\beta_{x_i}-\beta_*\right|} \tag{5}$$

where α and γ are parameters fixed by the user; σ_{xi}^2 is the variance of return of the selected portfolio and β_{xi} is the corresponding β; β_* stands for the β of the reference portfolio. The idea is to promote portfolios which have a small variance and a β close to β_*. This objective function is then aimed to take into account both the systematic and the specific risk.

The portfolio valuation function is transformed into the fitness function $\Phi(x_j)$ defined as:

$$\Phi(x_i) = \frac{F(x_i)}{\sum_{j=1}^{\mu} F(x_j)} \tag{6}$$

where $j = 1 \ldots \mu$.

The fitness function is used to select portfolios to generate new ones. Basically, differential evolution creates a new portfolio by adding the weighted difference between two portfolios to the third one. If the resulting portfolio yields a higher fitness function value than its originator, then the new portfolio replaces the old one; otherwise the latter is retained.

The selection of portfolio for the mating pool can be based on various criteria. In our system, the selection strategy is based on the roulette wheel [6; 13] in which the probability for a portfolio to be chosen is proportional to the value of the fitness function.

There are three stopping criteria in this algorithm. The first is defined by the acceptable level of valuation function value. The second is based on the homogeneity of population, defined as a minimal difference between the best and the worst portfolio:

$$\max_i(F(x_i)) - \min_j(F(x_j)) \le \varepsilon$$

where ε is a threshold defined by the user.

The third criterion is defined as a maximal number of generations. The algorithm stops when one of these three criteria is satisfied.

Several variants of portfolio generation operators are implemented in the system. They differ from the classical crossover or mutation operators. The principle is to perturb a chosen portfolio by adding weighted differences between more than one portfolio. Therefore, to distinguish these operators from classical ones, we call the „recombination by mutation”. The principle of recombination by mutation is shown in figure 3. For each portfolio x_i $j = 1, ..., \mu$ a new portfolio is generated according to:

$$x'_i = x_{S,i} + H * (x_{T,i} - x_{U,i})$$

with portfolios $x_{S,i}$, $x_{T,i}$, $x_{U,i}$ being different and $H > 0$ being an amplifier of the differential variation between the two portfolios T and U.

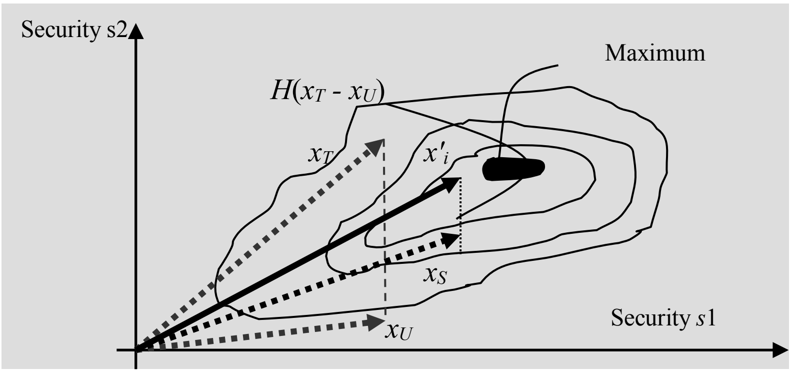

Fig. 3. Example of a portfolio perturbation by recombination by mutation

In our algorithm of differential evolution, the following recombination operators are designed:

$$x'_i = \begin{cases} x_{S,i} + H * (x_{T,i} - x_{U,i}) & (1) \\ x_{\max,i} + H * (x_{T,i} - x_{U,i}) & (2) \\ x_{\max,i} + H * (x_{T,i} - x_{U,i} + x_{S,i} - x_{V,i}) & (3) \\ x_i + \lambda(x_{\max,i} - x_i) + H * (x_{T,i} - x_{U,i}) & (4) \\ x_i & (5) \end{cases}$$

with $S, T, U, V \in [1, \mu]$ mutually different, $H > 0$, and $\lambda > 0$. The coefficients H and λ are given by the user.

The second operator is essentially the same as the first one, except that the portfolio to be perturbed is the best portfolio in the current population. In the third operator, the perturbation of the best portfolio results from four randomly selected portfolios. The fourth operator perturbs the portfolio by using two differences: the first being the difference between randomly chosen portfolios and the second being the difference between the best portfolio and one randomly chosen. The fifth operator simply makes a copy of the original portfolio.

The rate of operator application is variable during the evolution process. It depends on the quality of generated portfolios. In other words, the rate raises up if an operator improves the fitness value. This self-adaptive process has a significant impact on the efficiency of evolution.

The new population of portfolios is created by the replacement strategy. Two replacement strategies are built into the system:

$$S(\mu+\lambda) \rightarrow x_t = x_{t-1} \bigcup x'_t$$
$$S(\mu,\lambda) \rightarrow x_t = x'_t$$

where μ is the number of portfolios and λ is a number of offspring-portfolios with $\mu \geq \lambda$.

The first strategy, called $S(\mu+\lambda)$, selects the best μ portfolios among the *(μ+λ)* portfolios. The second one, called $S(\mu,\lambda)$, selects the best μ portfolios among the λ newly generated portfolios. To avoid losing the best portfolio from the previous generation, the elitist scheme can be applied. It is also possible to define the maximal number of generations during which a portfolio may „survive" through the time of evolution.

At the end of the process, the best portfolio for the next day, in terms of the fitness function, is returned as well as its characteristics. In the next section, differential evolution is illustrated using real market time series.

6. Case study: Portfolio evolution and simulation

The initial portfolio may be defined by a user or downloaded from the database. Once the portfolio is specified, the corresponding experts from the database can be selected. Experts are composed of the trading rules discovered by the genetic system [11]. For each day, according to its rules, a chosen expert generates a trading signal: buy, do nothing or sell. It should be noticed that expert trading decisions are respected in the portfolio evolution.

In our program, two modes of the portfolio optimization are implemented: day-by-day simulation and next-day prediction. In the paper, the first mode is briefly described in which the simulation algorithm executes a day-by-day optimization during a given period of time (in the example from the February 1, 2000 to February 15, 2000). During the simulation process the principle of self-financing is imposed. Figure 4 shows the parameters of simulation.

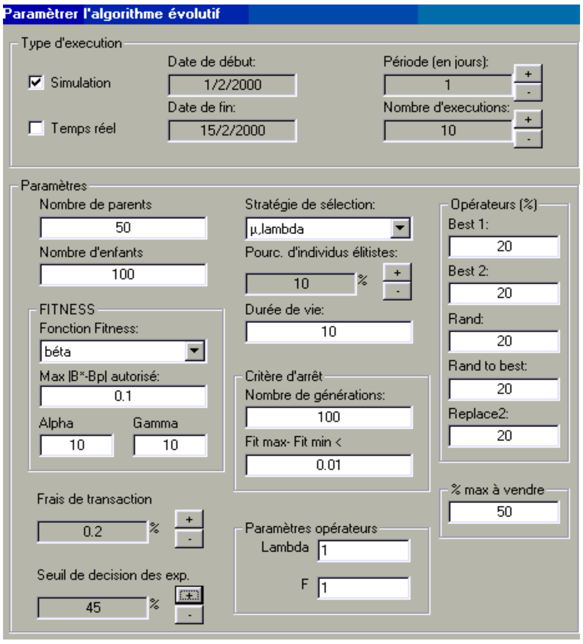

Fig. 4. Parameters of simulation

The parameters can be grouped into five classes which describe: the type of optimization, the process of evolution, the fitness function and constraints, the initial frequency of operators, and the expert decision thresholds. It is important to mention that the expert parameters have to be same as those used by the genetic algorithm during the expert discovery.

During the evolution, the system displays important information about the discovered portfolio, such as profit/lost, frequency of applied operators, minimal and maximal value of the fitness function in the population, etc. One can also visualize the evolution of portfolio composition.

Once the process of evolution is terminated, we can display a detailed report of the portfolio simulation (Fig. 5) and compare it with the passive buy-and-hold strategy.

The evolution report contains information about the portfolio wealth, the security values and expert decisions. This information can be completed by statistical characteristics of the final portfolio.

On the basis of the preliminary results obtained here, differential evolution appears to be a fairly robust and interesting portfolio optimization technique. The combination of the classical methods with the evolutionary approach seems to add new power. The approach can handle complicated portfolio optimization problems with multiple conflicting criteria. The convergence rate and the computational efficiency of the proposed method have been confirmed by the experimentation on forty

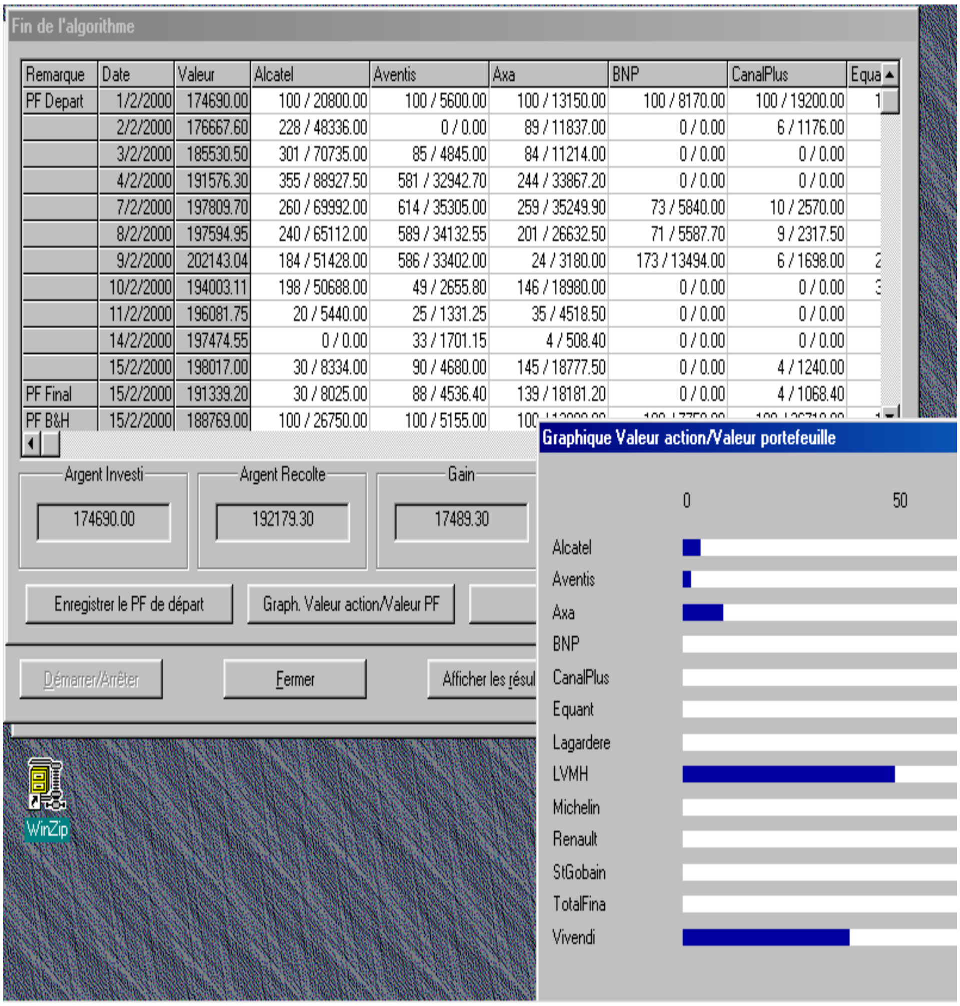

Fig. 5. Report of portfolio evolution

stock market time series (CAC 40). To formulate a more objective statement about the performance of differential evolution in portfolio optimization, more extensive testing is needed. On the other hand, more research on the portfolio encoding scheme and fitness functions is required to obtain more scalable results and to improve the presented software.

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OPTYMALIZACJA PORTFELA AKCJI METODĄ EWOLUCJI DYFERENCYJNEJ

Streszczenie

W artykule przedstawiono nowa metodę optymalizacji portfela akcji integrującą podejście klasyczne (oparte na wariancji stopy zwrotu i współczynniku β) z algorytmem ewolucji dyferencyjnej.

Idea optymalizacji opiera się na stochastycznej strategii zmian wektora akcji portfela sterowanej przez daną funkcje-kryterium. Zmiany wektora powodowane są przez zastosowanie operatorów genetycznych na składowych zbioru portfeli akcji. Decyzje tych zmian są generowane przez ekspertów utworzonych również w ewolucji genetycznej. W wyniku ewolucji populacji portfeli otrzymuje się optymalny portfel akcji na następny dzień. Symulacja ta, przeprowadzana dzień po dniu, pozwala na podjęcie właściwych decyzji o zakupie i sprzedaży akcji. W celu ilustracji strategii ewolucyjnej w artykule pokazano przykład działania programu na danych rzeczywistych pochodzących z giełdy paryskiej.